***Solve an Optimization Problem***

*WGU*

*Course Number: 605*

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**A: Identify the optimization problem in the given scenario**

**Business Need:**

Amazon Air has expanded its fleet and network to meet the growing demand for efficient, global package delivery. With two major hubs (Cincinnati and Fort Worth), three focus cities (Leipzig, Hyderabad, and San Bernardino), and 65 fulfillment centers, the company faces the challenge of optimizing its cargo distribution network. A key business need is to minimize the total shipping cost while ensuring customer demand is met, and hub/focus city capacities are not exceeded.

**Why Optimization Applies:**

This business need lends itself well to an optimization approach because it involves:

* A clear objective (minimize cost),
* Multiple constraints (hub and city capacity, center demand),
* And decision variables (quantities of cargo shipped between points).

Such characteristics are ideal for formulating a mathematical model, particularly a linear programming model, that can compute the most efficient distribution strategy.

**Linearity of the Problem:**

The optimization problem is linear. The objective function (minimizing shipping cost) and the constraints (capacity limits, demand satisfaction) can be expressed as linear equations or inequalities involving the decision variables. There are no non-linear relationships between variables such as exponents, products of variables, or conditional logic.

**Type of Optimization Problem:**

This is a linear programming transportation optimization problem. It falls under the category of network flow optimization, where the aim is to determine the optimal shipment quantities across a supply network to minimize total transportation costs while adhering to system constraints.

**B: Create Mathematical Representations of the Optimization Problem**

The mathematical framework for optimizing Amazon Air’s distribution network. The framework consists of an objective function that minimizes transportation costs, constraints that ensure operational feasibility, and decision variables representing shipment quantities between facilities.

**B1: Objective Function**

The primary objective is to minimize the total transportation cost of shipping cargo from two major hubs to three focus cities and 65 distribution centers. This cost depends on the volume shipped and the per-ton shipping cost between each origin-destination pair. Let:

* represents the tons shipped from hub i to focus city j,
* represents the tons shipped from hub i directly to center k,
* represents the tons shipped from focus city j to center k
* represents the respective per-ton shipping costs for those routes.

The objective function is as follows: Minimize Total Cost =

This function captures the full distribution cost across all allowed air routes in the network.

**B2: Constraints**

To ensure the distribution plan is feasible, the following constraints must be met:

* **Hub Capacity Constraints:** Each hub has a maximum monthly shipping capacity that cannot be exceeded:
* **Focus City Capacity Constraints:** Each focus city can receive up to a specified amount of cargo:
* **Focus City Flow Balance:** The cargo entering a focus city must equal the cargo exiting centers:
* **Center Demand Satisfaction:** Each center must receive exactly the amount of cargo needed:
* **Route Restrictions:** 189 specific route combinations are disallowed due to distance or feasibility:

These constraints ensure that the network adheres to real-world capacity, demand, and routing limitations.

**B3: Decision Variables**

The decision variables determine the amount of cargo shipped across each network segment. These variables are defined as follows:

* : Tons of cargo shipped from hub i to focus city j
* : Tons of cargo shipped from hub i directly to center k
* : Tons of cargo shipped from focus city j to center k

These variables must be non-negative, and some may be constrained to zero due to infeasible routes. They represent the core quantities Amazon needs to optimize to minimize total costs while satisfying delivery requirements.

**C: Describe the Approach You Will Use to Solve the Optimization**

A mathematical programming approach will be applied to solve the Amazon Air cargo distribution optimization problem. This approach leverages the structure of the problem, minimizing cost with linear constraints to find an efficient shipping strategy that meets operational and customer requirements.

**C1: Optimization Method or Algorithm**

The optimization method selected for this problem is Linear Programming. Linear programming is suitable because the objective function and constraints are all linear relationships involving continuous decision variables. Specifically, the problem is structured as a transportation problem, a subtype of LP focusing on minimizing the cost of shipping goods from sources (hubs and focus cities) to destinations (distribution centers).

Algorithms such as the Simplex Method or Interior Point Method can be used to solve the LP. These are well-established techniques in operations research that efficiently solve linear optimization problems even with hundreds of variables and constraints like those present in this case.

**C2: Tools and Technologies**

The following tools and technologies can be used to model and solve the optimization problem:

* **Microsoft Excel Solver** (for smaller test cases): Built-in optimization tool that allows quick testing of LP models by setting objective functions, decision variables, and constraints.
* **Python with PuLP or SciPy libraries**: Open-source libraries that allow defining LP problems programmatically and solving them using commercial or open solvers (like CBC or GLPK).
* **IBM ILOG CPLEX** or **Gurobi**: Industry-grade solvers known for their speed and ability to handle large-scale linear programs.
* **MATLAB** or **R (with lpSolve)**: Suitable for academic or technical analysis environments that require flexibility in modeling.

These tools enable precise definition of the model, efficient computation of the optimal solution, and scalability to handle all 189 variables and 73 constraints in the Amazon scenario.

**D: Asses the Risks and Limitation Involved in Your Recommended Optimization Approach**

While linear programming is a powerful tool for solving cost-minimization problems like Amazon’s cargo distribution network, it does have several risks and limitations that should be considered before full-scale implementation.

**Data Accuracy and Availability:**

The effectiveness of the optimization model heavily depends on the accuracy of the input data, including shipping costs, capacity constraints, and demand estimates. The resulting solution could be suboptimal or infeasible if these values are outdated, misestimated, or missing. Amazon must ensure that reliable and updated data feeds are available to support ongoing optimization.

**Static Assumptions in a Dynamic Environment:**

Linear programming models typically represent a snapshot in time and assume that all input values (e.g., costs, capacities, demand) remain constant. However, real-world logistics involve constant change due to fuel price fluctuations, aircraft availability, weather conditions, and unexpected delays. The static nature of LP may limit its responsiveness unless the model is updated frequently or integrated with real-time systems.

**Scalability and Complexity:**

Although LP solvers can handle large models, the number of variables and constraints in this problem (over 189 variables and 73 constraints) still introduces computational complexity. As Amazon’s logistics network grows or additional constraints (e.g., emissions, priority handling) are introduced, the model may become increasingly difficult to maintain or solve quickly, especially without commercial-grade optimization software.

**Exclusion of Qualitative Factors**

The model focuses solely on minimizing cost and does not account for other essential business considerations such as service level agreements (SLAs), regional delivery priorities, customer satisfaction metrics, or risk mitigation (e.g., spreading shipments across multiple locations to reduce disruption). These qualitative factors must be considered through complementary analysis or additional model constraints.

**Disallowed Routes and Model:**

With 189 disallowed combinations based on feasibility or distance, the model is subject to hardcoded constraints that may change due to geopolitical shifts, route openings, or temporary authorizations. This rigid structure may reduce adaptability unless the constraint logic is frequently reviewed and updated.

In summary, while linear programming provides a powerful and structured approach to solving Amazon’s distribution cost minimization problem, successful implementation will require careful data quality management, system adaptability, and real-world operational complexity.